Motional Feedback Theory in a Nutshell

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Contents

1 Introduction

This paper is meant to give some background knowledge that is used in the design of modern active controlled subwoofers.
It is based on the theory of motion control as presented in the book: "The Design of High Performance Mechatronics"[1]. For that reason the subject of motional feedback theory is treated in a limited way, concentrating on the typical stability requirements, error reduction and dynamics, that are related to the use of feedback of a loudspeaker. Although feedback is mainly addressed, also some words are spent to "Model Based Feedforward Control" as many people think that that will be the ultimate solution.

2 The Physical Meaning of Feedback

For those people who are reluctant with the use of mathematics, related to motion control, first the concept of feedback in a mechanical system is explained by means of how it changes the properties of a dynamic system.

2.1 Passive Feedback

In mechanical sense "feedback" relates to the application of a force, which counteracts a mechanical motion aspect, being a displacement, a velocity or an acceleration. In mechanics three dynamic elements are defined, which provide such feedback for each of the mentioned motion aspects. The first element, the spring, gives a counteraction force, which is proportional to the displacement between both ends of the spring according to Hooke's law. The second element, the damper, gives a counteracting force, which is proportional to the velocity between two sides of the damper. The third element, the mass, gives a counteracting force, which is proportional to the acceleration of an object. These feedback phenomena are called passive because they do not imply any supply of energy from outside to the system. It is useful to be aware of these intrinsic passive "feedback" properties of mechanical structures as these are directly comparable with the active feedback principles that will be described in the next section.

As explained in the paper on "Low Frequency Sound Generation by Loudspeaker Drivers" the dynamic behaviour of a loudspeaker driver is determined by the mass of the moving part, the membrane and actuator coil, the spring stiffness of the surround suspension and the air within the enclosure and the damping, mainly caused by the motion EMF and the low impedance of the amplifier. The frequency response of such a dynamic system is characterised by two frequency ranges, divided by a resonance frequency. Below the resonance frequency the system will move with an amplitude, which is proportional to the force and the inverse of the stiffness.

[1]The motion control section of the book is mainly written by Georg Schitter from TUVienna.

Above the resonance frequency the system will move with an amplitude, which is proportional to the force and the inverse of the mass. The resonance frequency is determined by the combined moving mass m and stiffness k in the following way:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

2.2 Active Feedback

Active feedback in motion control aims to mimic dynamic elements by measuring a mechanical aspect and supplying a force by means of an electronic amplifier and actuator, which counteract the measured aspect. This means that proportional position[2] feedback control creates a virtual stiffness, which drives the object to a wanted position. Proportional velocity feedback control creates a virtual damper, which reduces deviation of the velocity from a set value and proportional acceleration feedback control creates a virtual mass, which reduces deviations in the acceleration from a set value.

A remark must be made here on the term “proportional”. This indicates that there is no dynamic, frequency dependent relation between the measurement and the exerted force. Further on it will be explained that by differentiation and integration smooth transitions can be made between the different"“virtual” actively created mechanical elements by means of measuring only one of them.

With these findings it is easier to imagine the impact of feedback on the dynamic behaviour of a loudspeaker driver. Figure 1 shows the characteristic frequency response of a loudspeaker driver, when mounted in a closed-box enclosure. Below the resonance frequency it shows a +2 slope with 180°phase lead and above the resonance frequency it shows a flat response, because the radiated sound is proportional to the acceleration.

When applying proportional position feedback by measuring the position of the membrane and supplying a force that reduces the deviation from the wanted position, a virtual spring stiffness is added to the stiffness by the surround suspension and the enclosed air in the enclosure. As a result the resonance frequency will increase and the low frequency response is decreased. This is not a favourable situation and one of the first traps people fall into when thinking about active feedback of loudspeakers. Using position sensors is useless!

When applying velocity feedback by measuring the velocity of the membrane and supplying a force that reduces the velocity a damper is created. As a result the peak of the resonance is decreased, which is useful. In fact the combination of a voltage source amplifier with a loudspeaker driver creates a velocity feedback

[2]The use of the term “position feedback” or “position control” is more common instead of ”displacement control”. In fact displacement is the deviation from the wanted position, which is corrected by feedback. This wanted position can be stationary or changing.

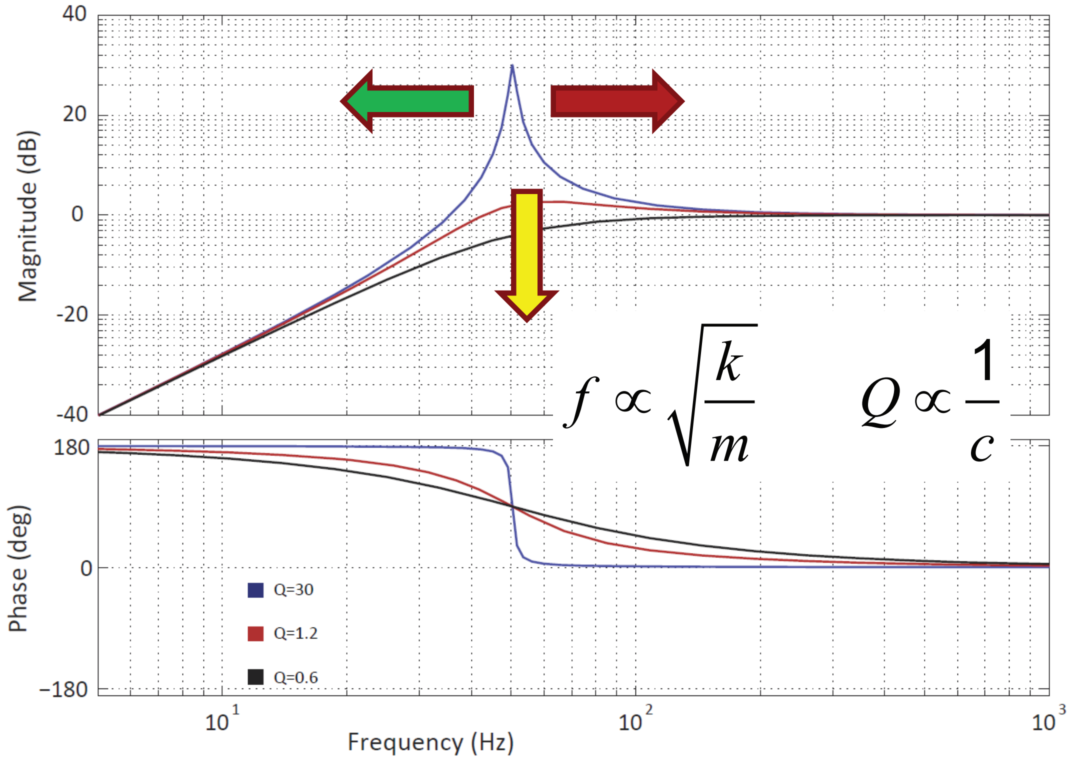

Figure 1: Proportional position feedback increases the stiffness, resulting in an increasing resonance frequency (red arrow). Proportional velocity feedback increases the damping, resulting in an decreasing peak at the resonance frequency (yellow arrow). Proportional acceleration feedback increases the mass, resulting in a decreasing resonance frequency (green arrow).

system because the motion EMF generates a force that counteracts the velocity. This principle is described in another paper on a Sensorless Velocity Feedback Subwoofer.

Finally, when applying acceleration feedback by measuring the acceleration of the membrane and supplying a force that reduces the acceleration a mass is created. As a result the resonance frequency will be lower, which is beneficial as that increases the frequency range with a flat response. It also decreases the output at higher frequencies and one might make the erroneous conclusion that it decreases the efficiency. This is not true because the added mass is virtual and the lower output is caused by a lower voltage signal at the loudspeaker terminals due to the feedback. This means that this effect can be compensated by increasing the input signal again (see the footnote at page 9.

In the following this more qualitative describing explanation of the principle of feedback is shortly presented in a more official “control-technology” way.

3 Feedback Control Loop

Figure 2 from the above mentioned book shows a basic feedback control loop of a motion system.
The **plant** is a control engineering term for the physical motion system that needs to be controlled. In a motional feedback system it consists of the power amplifier, actuator, and the mechanical structure of the moving parts of the driver, the actuator coil, cone, rubber surround and spider.
In a motional feedback system the **Input disturbance** is the noise that is generated in the controller. It will be addressed in the design chapter.
The **Process disturbance** refers to nonlinearity and noise from the mechanics like rubbing and leaking holes. Nonlinearity causes distortion and should be kept minimal or reduced by feedback and the other disturbance sources should be avoided as hissing leaking holes are irritating, while rubbing implies wear.
The **Output disturbance** represents the influence from the environment, like the sound pressure from another loudspeaker. When using two subwoofers for the same signal this mutual disturbance is in fact beneficial as it increases the total efficiency at low frequencies[3].
Finally and most importantly the **Sensor disturbance** represents the measurement error by the sensor. In a feedback controlled loudspeaker it consists mainly of thermally induced $1/f$ noise and non-linear distortion. It is the most important source of disturbances as the controller will force the motion system to follow the erroneous measurement.

3.1 Interaction of Elements

Each of the elements in the feedback control chain has its own inherent dynamic properties. They also interact in both directions in such a way that each element not only determines the input of the next element but also influences the previous element by its dynamic load. The best example for this is the interaction between the actuator and the amplifier.
First of all the current from the amplifier will generate a force in the actuator following Lorentz' law, which can be written in two ways:

$$F = BI\ell \quad \text{or} \quad F = I\frac{\mathrm{d}\Phi}{\mathrm{d}x} \tag{2}$$

where:

- B = the flux density (T) of the magnetic field at the coil
- I = the current (A)

[3]Too few people are aware of the fact that an uncontrolled loudspeaker, so without a connected and working amplifier, will act as a resonator for sound coming from other loudspeakers! One should always short out the unused loudspeakers when judging and comparing different loudspeakers.

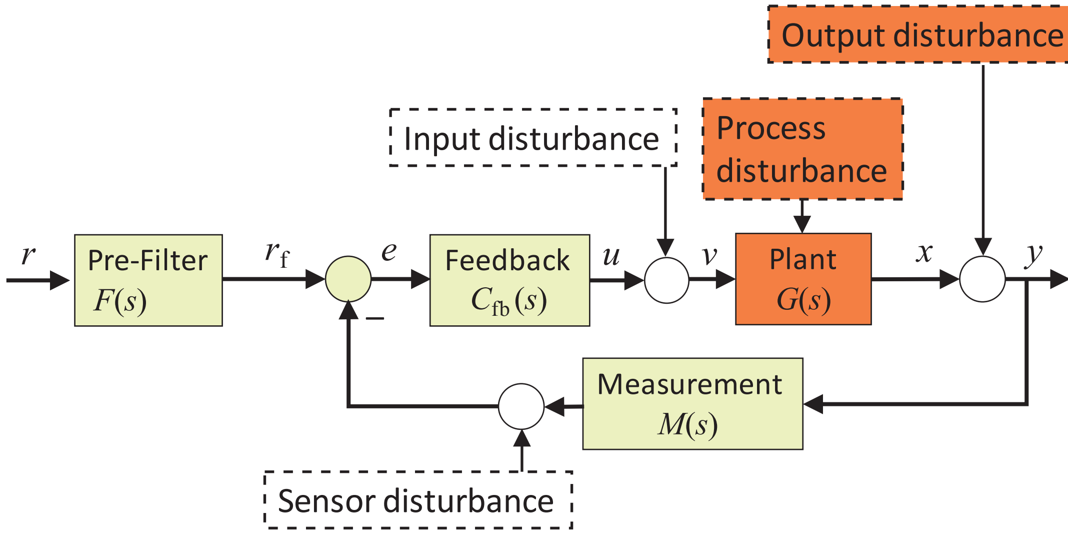

Figure 2: Block diagram of a control system with feedback control. The plant consists of the moving parts of the loudspeaker driver. The diagram clearly shows the different places where interfering disturbances impact the system. The indicated variables are commonly used symbols in control engineering.

- ℓ = the total length of copper wire of the coil in the magnetic field
- $\frac{d\Phi}{dx}$ = the change in flux linkage, the amount of flux captured by all windings of the coil together.

Both notations are in principle correct and the first notation is best known, while the second is better as it prevents errors in designing actuators. For this report the first notation is sufficient, while also driver manufacturers use the term $B\ell$, also called the "Force Constant", as one of the relevant parameters for a driver.

The above implies that the amplifier makes the moving part of the driver move by its current.
As a reaction a voice coil in a magnetic field will generate a voltage V_m, which is proportional to the force factor $B\ell$ and the motion velocity v_m:

$$V_m = B\ell \frac{dx}{dt} = B\ell v_m \tag{3}$$

A voltage source amplifier, as is used by all audio brands, will short circuit the coil for this motion induced voltage V_m. Due to this short circuit V_c will generate a current in the coil, which in its turn generates a force according to Equation (2) that is proportional to velocity v_m and works in the opposite direction of the movement. This means that the use of a voltage source amplifier causes a damping effect, which is required for a non-controlled loudspeaker as otherwise the first resonance would have a very high Q value, where Q represents the level of the resonance above the non resonant response. The effect for different levels of damping is shown in Figure 3 and the shown peaking in output is very well audible.

As a conclusion it is clear that the amplifier influences the dynamic behaviour of the driver.

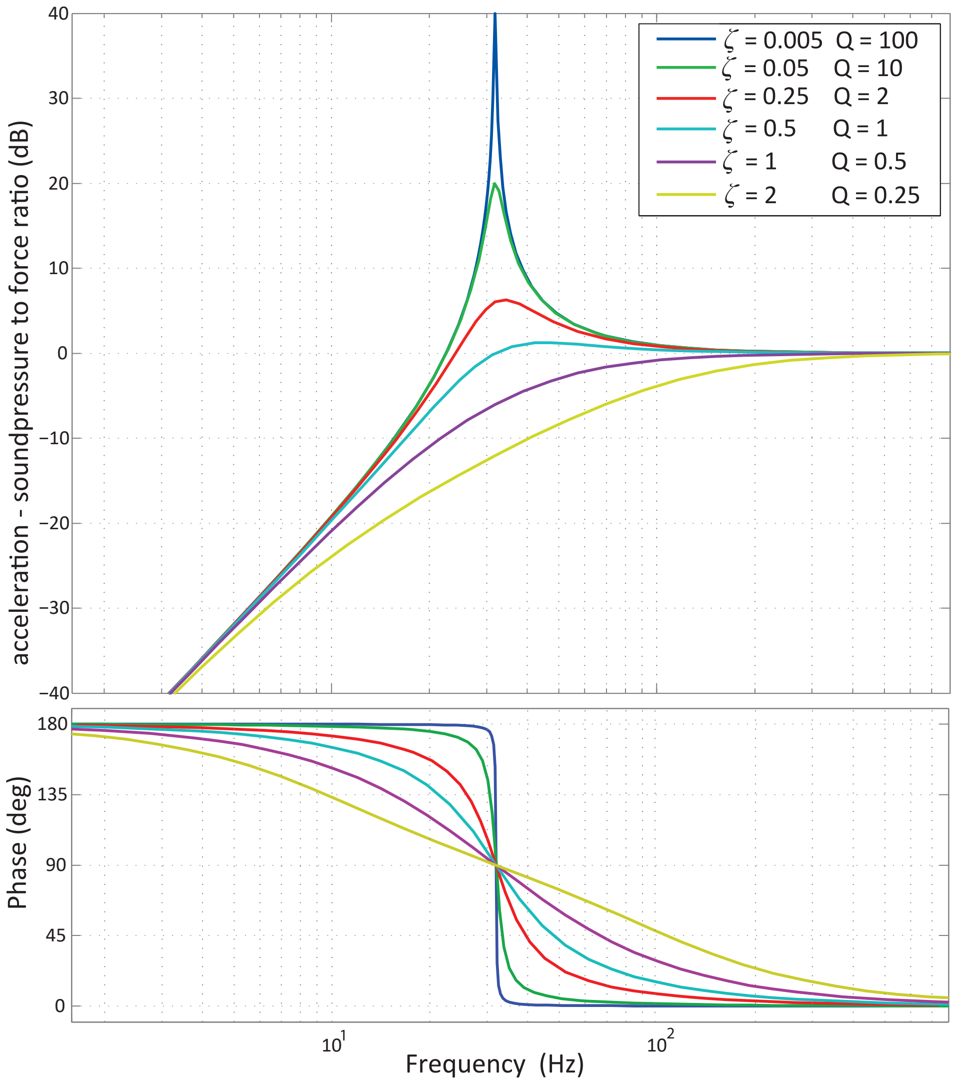

Figure 3: Frequency response of the radiated sound of an electrodynamic loudspeaker, normalised to 0 dB above the first resonance frequency with different damping settings.

3.2 Properties of Feedback Control

In feedback control the actual status of the moving parts is monitored by a sensor and the controller is generating a control action based on the difference between the desired motion (reference signal) and the actual motion (sensor signal).
The output y as shown in Figure 2 is measured and compared with (subtracted from) r_f, which is the reference r after input filtering. The result of this comparison, which is the error e, is used as input for the feedback controller, which tries to keep this error as small as possible.
Feedback control is also called *closed-loop* control, because the sensor signal is fed back in a closed-loop to the input of the system.
In control theory the mathematics of the Laplace transform are used to model the dynamic behaviour of a mechanical system in the frequency domain, based on the equations of motion in the time domain. Working with the resulting frequency

responses is more easy to work with as controllers can be made by simple filters. By using the Laplace variable $s = \sigma + j\omega$, equations called "Transfer Functions" are derived from the equations of motion, which enable to generate Bode-plots, showing the frequency response in both amplitude and phase. Using the defined notions from Figure 2, the transfer function of a feedback loop is derived from the following equations starting with the error e:

$$e = r_\mathrm{f} - M(s)y, \quad y = G(s)C_\mathrm{fb}(s)e, \quad \Longrightarrow \quad T(s) = \frac{y}{r_\mathrm{f}} = \frac{G(s)C_\mathrm{fb}(s)}{1 + M(s)G(s)C_\mathrm{fb}(s)} \tag{4}$$

Including the input filter the total transfer function of the feedback loop from the reference signal r to the output y as shown in Figure 2 is given by:

$$T(s) = \frac{y}{r} = \frac{G(s)C_\mathrm{fb}(s)}{1 + M(s)G(s)C_\mathrm{fb}(s)} F(s) \tag{5}$$

When considering that $G(s)C_\mathrm{fb}(s)$ is the gain of the forward path from error to output and $M(s)G(s)C_\mathrm{fb}(s)$ equals the forward gain times the sensor gain and leaving away the (s) terms after the equal sign for reasons of simplicity the transfer function can be written as[4]:

$$T(s) = \frac{y}{r} = \frac{\text{forward gain}}{1 + M \cdot \text{forward gain}} F \tag{6}$$

by dividing the numerator and denominator by the forward gain it becomes clear that a high gain in the forward path will cause the transfer function to be only dependent of the input filter and the sensor:

$$T(s) = \frac{y}{r} = \frac{\text{forward gain}}{1 + M \cdot \text{forward gain}} F = \frac{1}{\frac{1}{\text{forward gain}} + M} F \approx \frac{F}{M} \tag{7}$$

In control design one has the freedom to choose the prefilter $F(s)$, the sensor $M(s)$ and particularly the controller $C_\mathrm{fb}(s)$ such that the total transfer function fulfils the desired specifications.

With these properties feedback control has the following benefits for motional feedback of loudspeakers:

- **Reduction of the effect of disturbances**: Disturbances of the controlled motion system like distortion by non-linearity and unwanted sounds by undamped resonances are observed in the sensor signal, and therefore the feedback controller can compensate for them.
- **Handling of uncertainties**: Feedback controlled systems can also be designed to cope with changes and tolerances in the different properties of the elements. This is called **robustness**, which means that the stability and performance requirements are guaranteed even for parameter variations of the controlled mechatronic system.

[4]These equations show that feedback reduces the response of the loudspeaker driver (which is included in the "forward gain") by the terms in the denominator. This effect on the forward gain can be compensated by increasing the gain F of the pre-filter.

Although feedback control provides some very good features, it has of course also some pitfalls that have to be dealt with:

- **A good sensor is required**: The feedback loop is closed, based on information from a sensor. Therefore feedback control only can be as good as the quality of the sensor signal allows. In precision positioning systems accurate sensors are required with high resolution and bandwidth, which are very costly. The measurement and sensing system often takes a substantial part of the total financial budget.
- **Limited reaction speed**: A feedback controller only reacts on errors, differences between the reference signal and the measured system status, which means that the error has to occur first before the controller can correct for it. Without an error there is no output!
- **Feedback of noise**: As mentioned earlier, by closing the loop, the sensor noise is also fed back, causing the sound to follow the noise instead of only the wanted reference input signal.
- **Can introduce instability**: When the "negative" feedback becomes positive by phase delays in the loop the feedback system can (and will!) become unstable, thereby causing the system to resonate it its maximum power at a low or high frequency. Due to the continuous maximum power it can eventually destroy the driver!

Feedback control is a very useful principle in loudspeakers for low frequencies as it reduces distortion and resonating effects as long as one takes precautions agains instability..

3.3 Stability and Robustness in Feedback Control

[5]As mentioned in the last bullet of the previous section one should always consider phase relations when applying feedback. With loudspeakers two areas in the frequency range are giving problems. At high frequencies the natural inertia (slowness) of things will cause phase delays of which many are unavoidable. This poses a limit to the maximum frequency at which feedback can be applied with success. With precision positioning systems as are presented in "the book" this is the only frequency range of phase problems because these systems should operate from DC. A loudspeaker however has also a limitation at DC due to the fact that the sound is proportional to the acceleration. This means that a loudspeaker is not able to generate DC sound. Fortunately that is not even required but the fact that the frequency response shows a decline at lower frequencies automatically has impact on the phase. Where at higher frequencies the phase is lagging behind (delay) at

[5]The following is also partly copied from "the book" and adapted to motional feedback.

lower frequencies the phase is advancing (leading). Both can cause the phase to change more than 180°, changing negative feedback into positive and creating an unstable system.

Between these extreme frequency ranges it is necessary to create a maximum forward gain in order to reduce the errors.

The challenge in designing a controller for motional feedback is thus to optimise between a high loopgain at the frequencies that have to be controlled and a low loopgain at the other frequencies. The optimal tuning of a feedback loop is called **loop-shaping design**.
The most important and characteristic frequency area for the analysis of a controlled mechatronic system is around the open-loop unity-gain cross-over frequency, as shown in Figure 4 for the upper frequency limit. In a motional feedback system the closed-loop *bandwidth* is directly related to the unity-gain cross-over frequency as above this frequency the loop gain becomes smaller than one and consequently the feedback controller becomes no longer effective. Usually the term bandwidth

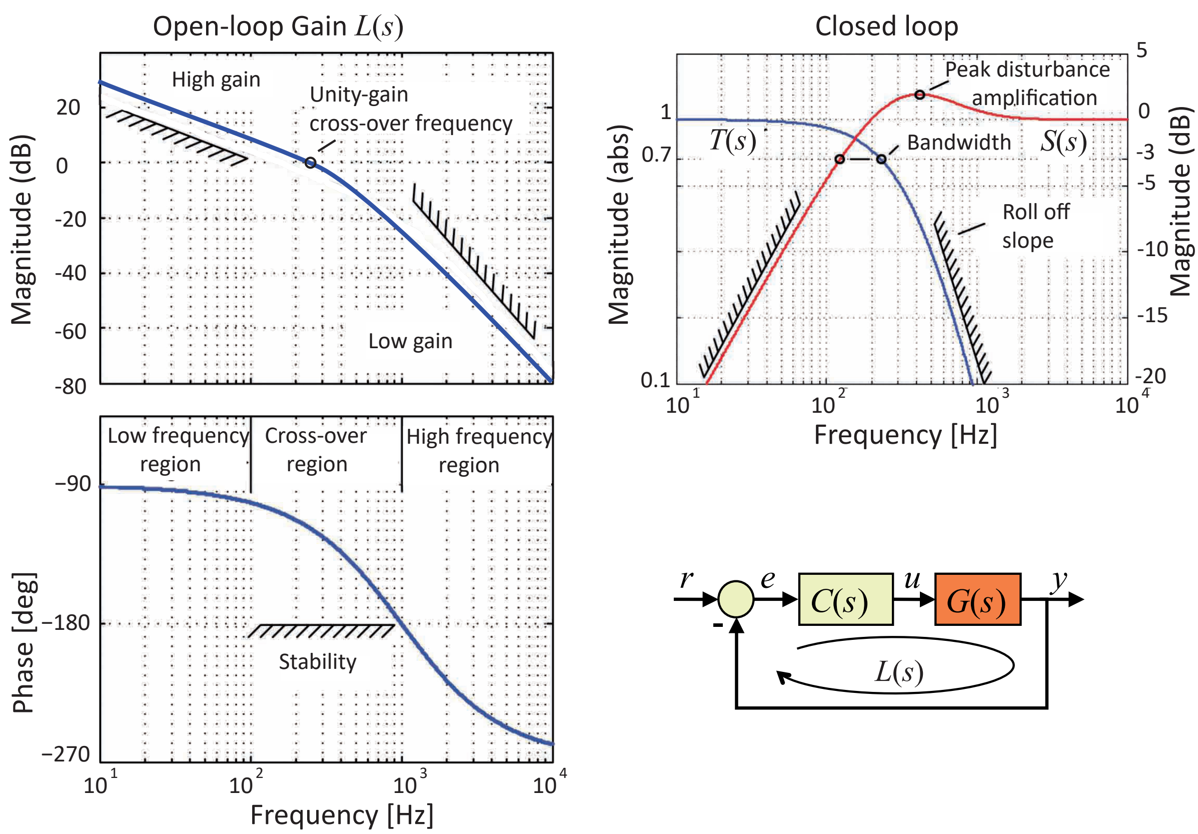

Figure 4: Stability condition and robustness of a feedback controlled system for the high frequency limit. L(s) equals the loop gain (forward gain times M(s), where M(s) is assumed to be equal to one in this figure), T(s) is the closed loop response and S(s) is the Sensitivity. The desired shape of these curves guide the control design by optimising the levels and slopes of the amplitude Bode plot at low and high frequencies for suppression of the disturbances (sensitivity) and of the phase Bode plot in the cross-over frequency region.

is defined as the frequency band where the power of the output signal of a system becomes less than half the desired power level. In terms of signal amplitude the corresponding value is equal to $1/\sqrt{2} \approx 0.7$. In decibels this value is equal to -3 dB and this value is a well-known definition for the bandwidth of filters and loudspeakers. In the context of reduction of errors it is preferred to define the control bandwidth as the range between the low and high unity-gain cross-over frequencies, where the amplitude of the **open-loop** frequency response exceeds a value of one. It is this open-loop gain that determines the suppression of disturbances, also called “Sensitivity” which is equal to the following equation.

$$e(s) = \frac{1}{1 + M \cdot \text{forward gain}} \tag{8}$$

When used in full detail using mathematical modelling software like MATLAB it will show that with systems of a higher order than one (which is the case with motional feedback) the sensitivity is increased at the frequency area just above the unity gain cross over frequency. This effect is called the Bode-integral theorem and cannot be avoided. It is a sacrifice for the error reduction at the frequencies where the loopgain is much higher than one. The only way to keep the effect small is to spread it over a larger range by optimising the phase and amplitude margins, which are explained in the following.

The key condition for closed-loop stability is that the total phase-lag for high frequencies and phase lead for low frequencies of the open-loop system, consisting of the feedback controller in series with the mechanics, must be less than 180° in the frequency region of the cross-over frequencies. A system that has exactly 180° phase-lag at the cross-over point is called marginally stable. In this situation the smallest additional time-delay or phase-lag would make the closed-loop system unstable.

Even though most audio designers hardly use it, a Nyquist plot, like the example shown in Figure 5, is most suitable to analyse the robustness of a feedback system. It is an analysis tool that examines the **open-loop** frequency response of the feedback system including phase to predict the stability and the **closed-loop** response after the loop is closed. Its use is based on the Nyquist stability theorem, stating that a closed loop system will be stable when the Nyquist plot of the open-loop transfer function does not show a net clockwise encircling of the -1 point on the real axis. In other words a stable system after closing the loop is recognised in the Nyquist plot when the -1 point on the real axis is kept at the left-hand side upon passing with increasing frequencies.

The complexity of the plot is in the fact that it is a complex plot with real and imaginary axis where phase and magnitude are combined and the frequency axis is not clearly shown. It is a 2 dimensional vectorial plot where the distance from the origin indicates the gain and the angle of the line through the vector point and the origin with the right horizontal (positive real) axis determines the phase. An angle rotating clockwise is a negative phase relation and counterclockwise indicates a positive phase. For all frequencies such a vector point can be constructed and by connecting these points for all frequencies a curve is created (the blue line) along

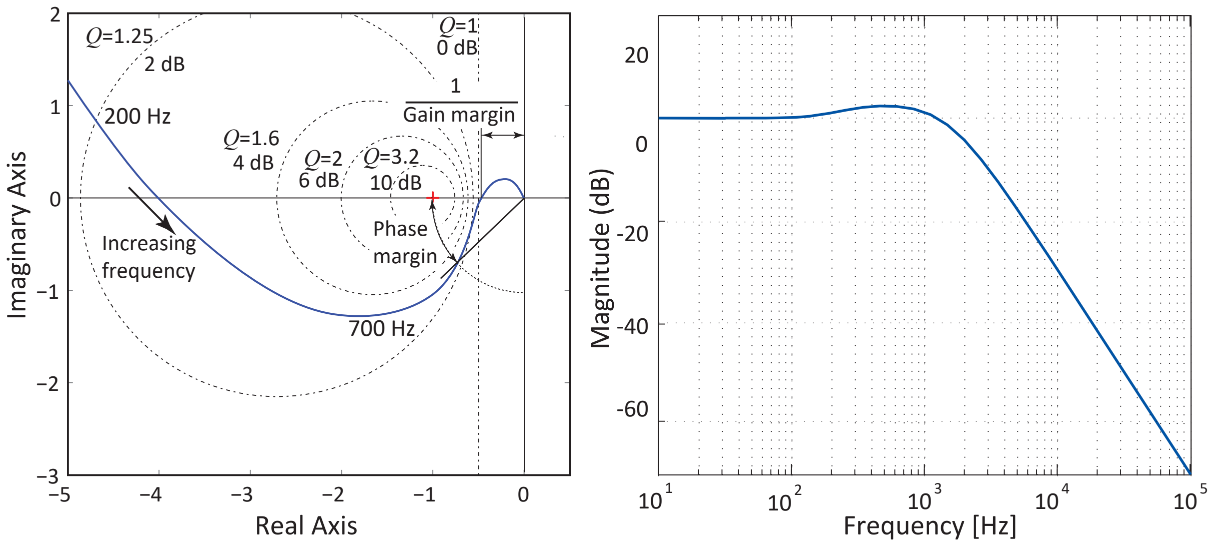

Figure 5: The Nyquist plot of the open-loop response of a feedback system and its corresponding closed-loop frequency response of an example with HF limitation only. Stability is guaranteed when the −1 point on the real axis of the Nyquist plot is kept at the left-hand side of the open-loop response-line upon passing with increasing frequency. The dashed circles at the left graph determine the magnitude peak (Q) of the frequency response after closing the loop at the frequencies where the response-line crosses the circles. In this example +2 dB @ 200Hz and +3 dB @ 700 Hz.

which the frequencies could be noted. An arrow alongside the blue line indicates increasing frequencies. Because both a phase of >+180 degrees and <-180 degrees is indicating potential trouble, most of the plot shows the left half from the origin. The fact that the frequency is not noted is overcome in practice because its first purpose is to show potential problems by means of computer simulation. A Nyquist plot is never made by hand while the modelling software immediately indicates the frequency, when pointing with a mouse to a place on the curve.

The stability analysis with a Nyquist plot is done by examining the distance and direction of the plotted response graph of the **open-loop** system relative to the location of the −1 point on the real axis. The graph shows margin circles related to the capability of the **closed-loop** system to follow a reference input signal. Two values are shown in the Nyquist plot that are related to the robustness for stability of the closed-loop feedback system, the *gain margin* and the *phase margin*.

The gain margin determines by which factor the open-loop gain additionally can increase before the closed-loop system goes unstable. It is defined by the distance between the loop-gain $L(s)$ and unity-gain at the frequency where the phase-lag of $L(s)$ becomes more negative than −180°. The gain margin can have values between one and infinite. With first- and second-order transfer functions where the phase does never become more negative than −180° the gain can be increased theoretically to infinite, corresponding to an infinite gain margin.

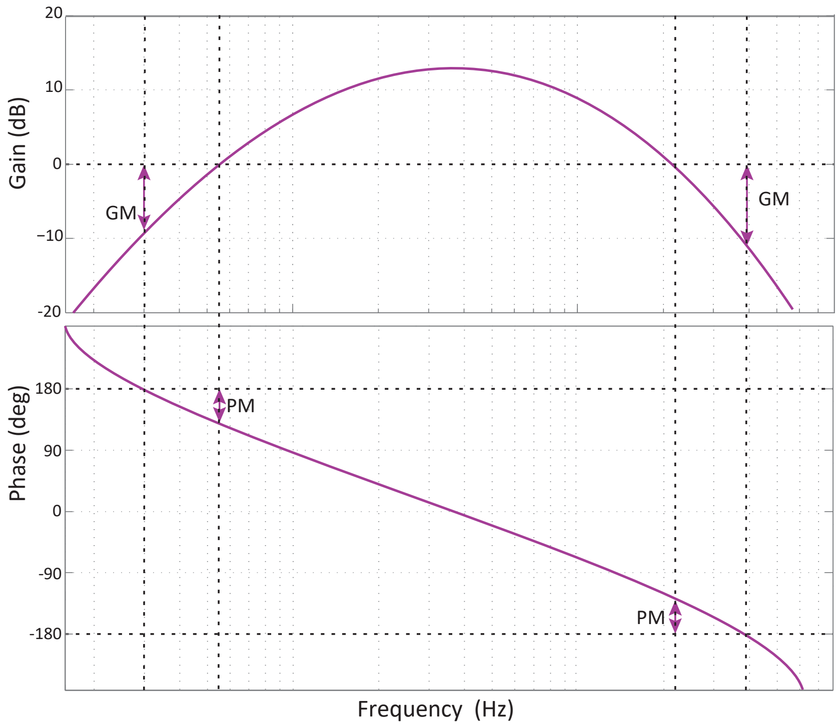

Figure 6: The Gain (GM) and Phase (PM) Margin in the Bode plot. At the LF bandwidth limit the phase-lead should be less than +180° at 0dB gain, while the gain should be below 0dB at +180° phase. At the HF bandwidth limit the phase-lag should be less than -180° at 0dB gain and the gain should be below 0dB at -180° phase.

The phase margin determines how much additional phase lag at the unity-gain cross-over frequency is acceptable before the closed-loop system becomes unstable. It is defined by the difference between the actual phase-lag of $L(s)$ and -180° at the unity-gain cross-over frequency.

When looking at the shown Nyquist plot the modelled example shows a phase that becomes more negative than -180 degrees at lower frequencies. This is counterintuitive but is OK as with increasing frequency the -1 point stays at the left hand side. If however the gain of one element in the feedback loop is **reduced** with more than a factor four the -1 point will be passed at the right side and the system will become unstable. This situation is called “conditionally stable” and is to be avoided when the gain can vary in the controlloop, as is the case with drivers with a large excursion range, like in subwoofers.

For an unconditionally stable system it is fortunately often sufficient to analyse the stability of the feedback loop by means of only the frequency and phase responses in the Bode plot, as shown in Figure 6. As long as the phase margin at both ends of the open-loop bandwidth is in the order of 45° or more and the gain margin is in the order of 6dB (factor 2) or more, a perfectly stable tuned feedback system is obtained.

4 Model Based Feedforward Control

Most of the actual research in industry and academia aims for "sensor-less" control of a loudspeaker by means of "Model-Based Feedforward Control" (MBFC). Indeed this is one of the most important research fields in the high-tech industry for one important reason: Feedback needs an error to act anyway, so it is a reaction with an unavoidable delay, the "settle time". In high-tech mechatronic motion control it is a design rule to first compensate any error sources and only apply feedback for real unknown errors.

MBFC starts with the idea that as long as one knows exactly how a system works and the system behaves reproducible, it is possible to control it by modifying the input signal in such a way that it compensates the deviations that are cause inside the (loudspeaker) system. This compensation is easiest explained in mathematics. Assume $\boldsymbol{y}$ is the output of a system in reaction to the input $\boldsymbol{u}$:

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{u}, \tag{9}$$

where $\boldsymbol{C}$ is equal to the process of the system, which incorporates errors. If you desire output $\boldsymbol{y}$ you only have to supply the system with $\boldsymbol{y}$ times the inverse of $\boldsymbol{C}$ ($\boldsymbol{u} = \boldsymbol{C}^{-1}\boldsymbol{y}$) as then the output will be:

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{u} = \boldsymbol{C}\boldsymbol{C}^{-1}\boldsymbol{y} = \boldsymbol{y} \tag{10}$$

Although this looks trivial, it requires the process $\boldsymbol{C}$ to be invertible, which means that one has to be able to derive the input from the detected output. In mathematics there are methods to see if a process is invertible, its matrix $\boldsymbol{C}$ should be square with a non-zero determinant. It goes, however, too far to do that here in this paper for a loudspeaker. It is more easy (and I know the theoretical people will not be pleased) to look at the phenomena that play a role, the distortion sources as described in the paper "Distortion Sources in Subwoofers", the position dependent gain, the current dependent reluctance force, the temperature dependent resistance and the non-linear selfinducance. For compensating (inverting) the position dependent gain one has to know the position of the membrane, which can only be derived by means of the dynamic model of the system with the input current. For low frequencies the dynamic model only consists of the mass of the moving diaphragm with coil and the total stiffness. It seems not too difficult to make the calculation, as long as the system does not change over time. An error in the model, like a shift in the fundamental resonance frequency will easily cause the compensation to work out of phase with the problem, thereby increasing it rather than solving it. Such a shift is well possible for instance due to the influence of temperature, which amongst others changes the stiffness of the surround. The reluctance force is also position dependent so when necessary to compensate it one needs to know the position with the same risks for errors as with the gain. The temperature of the coil can be calculated from the current that passed over time and a previously determined thermal model of the system. And finally one has to derive a good model for the non-linear selfinductance,

which can be sample dependent, because of the differences in magnetisation by the permanent magnets. Indeed it is possible to do this for one loudspeaker that has been measured and modelled but in the audio field the production of loudspeakers is not very strict with large tolerances on especially the magnetic part. It would only work when the system could be regularly calibrated by means of a suitable.....sensor!!!
In that case one might wonder why not use the sensor for active real-time feedback. The main drawback is then that one needs a real time sensor and these are expensive and critical with noise.

4.1 Adaptive Learning Feedforward and Observer

The use of digital controllers with ample memory opened up the possibility to learn from previous errors, similar to the motion control of a human being. Adaptive Feedforward Control (AFC) is well applicable in repeating actions and there is one thing for sure, A loudspeaker membrane is continuously repeating its motions. The repeating action allows for averaging the sensor signal, which reduces the impact of noise.
Still, due to all dynamic effects and non-linearities the behaviour is different for many small frequency areas while not constant over time. With modern control however an intermediate solution is possibly applicable using a real-time *estimator*, which is based on the model of the plant including the eigendynamics. Such an estimator is also called an *observer* while it observes the behaviour of a system by comparing it with the modelled behaviour and correcting its model parameters on this comparison in a process called *innovation*. Such an observer also allows a trade-off between the bandwidth (speed) of the estimation and the noise performance. An observer with an optimal trade-off between these two important properties is called a *Kalman-filter*, named after the Hungarian mathematician and electronic engineer Rudolph Emil Kálmán.
Figure 7 shows the configuration of an observer in combination with state-feedback[6] control of the observed system. The blocks in the dashed box represent the real mechatronic system. The blocks in the dotted box represent the mathematical model, which is implemented on a computer to simulate the behaviour of the mechatronic system in real-time. When both systems receive the same input signal $\boldsymbol{u}$, and both systems are identical, which means that a perfect model is available, both outputs $\boldsymbol{y}$ and $\boldsymbol{y}_\mathrm{o}$ should be the same. However, in reality always modelling errors will occur while also the mechatronic system can be disturbed by external forces that are not taken into account and causing position and velocity errors. To compensate for these deviations the observer-gain matrix $\boldsymbol{L}$ is introduced, which determines the innovation process by feedback of the prediction error to the observer, given by the difference between the output of the model and the output of the real system ($e = \boldsymbol{y}_\mathrm{o} - \boldsymbol{y}$). $\boldsymbol{L}$ has to be designed such that the closed-loop system for the observer

[6]State feedback is a mathematical method to design a discrete time digital controller. For more info see the book “The Design of High Performance Mechatronics”

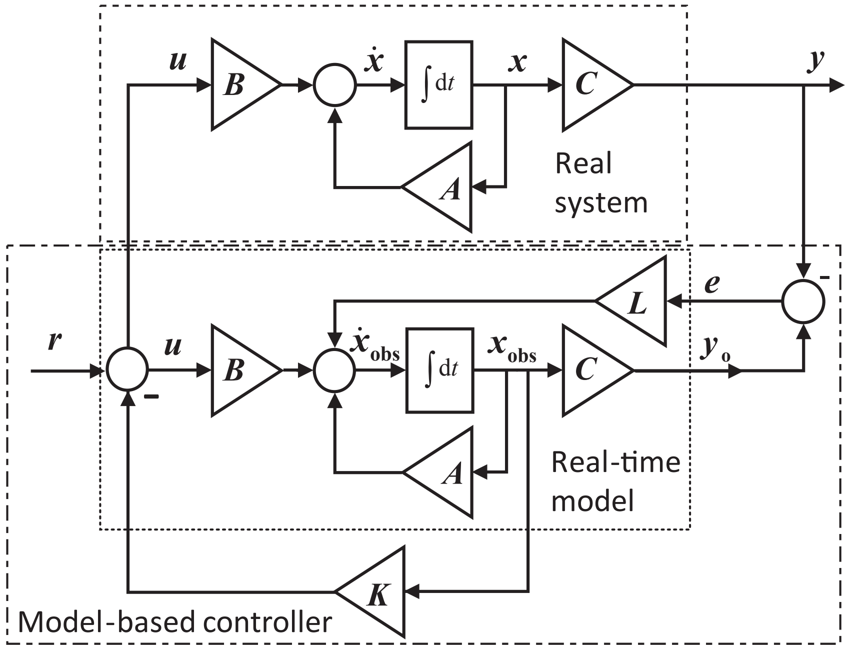

Figure 7: Model-based controller with an observer to estimate not measured values in a control system. The real-time feedback path is determined by the feedback matrix $\boldsymbol{K}$ based on estimated values from within the model. The model is updated by the difference between the observer output $\boldsymbol{y}_0$ and the real system output $\boldsymbol{y}$ via the matrix $\boldsymbol{L}$.

part is stable.

In spite of these new methods up till now no loudspeakers applying these technologies are developed, as far as the author knows. The main reason might be that it still requires a sensor, while real time feedback is proven to be a highly successful approach as applied by RMS Acoustics & Mechatronics.